PROBLEM # 678

Posted on:10 November
Due on:17 November

The twelve edges of a cube are assigned the integers from 1 to 12. The assignment has been made in such a way that no two edges have the same integer. Each of the eight vertices is concurrent with three edges. Let us call the weight of a vertex the sum of the three integers assigned to the concurrent edges of the vertex. Is it possible to assign the integers to the edges so that all the vertices have the same weight? If so, give such an assignment. If not, explain why.

The problem of the week can be found online at

`http://potw.mth.cmich.edu/`

Solutions can be mailed to

chakr2d@cmich.edu

with subject line "POTW 678"

Solution to Problem # 678

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Solution. No such assignment is possible.

Suppose, by way of contradiction, that all vertices have weight a. Note that a must be an integer. There are eight vertices on a cube, so the sum of all the weights is is $8a$. On the other hand, each edge is counted twice, so the sum of the weights is

$$2(1+2+3+\ldots+12) = 12 \cdot 13 = 156.$$

So we have $8a = 156$, or

$$a = \frac{156}{8} = \frac{39}{2}.$$

$39/2$ is not an integer, so we have a contradiction.

□

Problem 678

Correct solutions were submitted by Isabella Tucker, Jason Peterson Nicholas Stanlis, Ruvarashe Musasiwa, Lane Sirois, Carter Moleski and Kendal Swatosh. Based on a draw of lots, the prize is divided between Musasiwa and Swatosh. Partial solutions were also submitted by Claudia Mapes, Sydnee Renee Allen and Sungeun Kim.