Solution to Problem #674

Problem: In the city of Corruptville, politicians never tell the truth and nonpoliticians always tell the truth.

1. A stranger meets Alice, Bob, Chuck, all of them from Corruptville and and asks Alice, "Are you a politician?", and Alice answers the question. Bob says that Alice denied being a politician. Chuck says that Alice is a politician.

How many of the three people are politicians?

2. The stranger now meets three other natives of Corruptville: Dave, Ed and Felix, and asks them, "How many of you three are politicians?" Dave says, "We are all politicians". Ed says, "No, just two of us are politicians". Felix then says, "That isn't true either."

Is Felix a politician?

Solution. 1. If Alice is a politician, then she denies it, and, therefore, Bob and Chuck tell the truth, so they are not politicians. If Alice is not a politician, then she says so. Therefore, Bob again tells the truth, while Chuck lies. Thus, Bob is not a politician, while Chuck is. In either case, there is one politician among the three.

2. If Ed is a politician, then he lies, so either (1) all three are politicians, or (2) Ed is the only politician. In case (1), Felix tells the truth, which contradicts the assumption that all three are politicians. In case (2), the Dave lies, which contradicts the assumption that Ed is the only politician. Therefore, neither of the two cases is possible, which means that Ed is not a politician. Therefore, he tells the truth, i.e., there are two politicians among the three natives. Therefore Dave and Felix are politicians.