Problem: Let $t = cos(20^\circ)$. Show that there is a nonzero polynomial P with integer coefficients such that P(t) = 0.

Solution. Let $x = 20^{\circ}$, so that $t = \cos x$. Notice that

$$\cos(3x) = \cos(60^\circ) = \frac{1}{2},$$

and for any angle x we have the triple-angle identity

$$\cos(3x) = 4\cos^x - 3\cos x,$$

so that we have

$$4t^3 - 3t = \frac{1}{2}.$$

Consequently, we have P(t) = 0, where P is the polynomial with integer coefficients

$$P(t) = 8t^3 - 6t - 1.$$

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