PROBLEM # 679

Posted on:17 November
Due on:24 November

A perfect square number has four digits. When each digit is increased by 1, another perfect square is formed. What are the two perfect squares?

The problem of the week can be found online at

`http://potw.mth.cmich.edu/`

Solutions can be mailed to

chakr2d@cmich.edu

with subject line "POTW 679"

Solution to Problem # 679

Problem: A perfect square number has four digits. When each digit is increased by 1, another perfect square is formed. What are the two perfect squares?

***Solution*.** When 1 is added to each digit of a four-digit number, the number is increased by 1111. Thus we are looking for two squares, say m^2 and n^2, that differ by 1111. In other words, we are seeking solutions of the equation $m^2 + 1111 = n^2$ in which m and n are positive integers. The equation may be written as $n^2 - m^2 = 1111$. By factoring the left-hand side of this equation, we see that it can be rewritten as $(n - m)(n + m) = 1111$. So $n - m$ and $n + m$ are positive integers that are factors of 1111 and whose product is 1111. Also $n - m < n + m$. 1111 can be written as the product of two such positive integers in just two ways, namely as 1×1111 and 11×101 If $n - m = 1$ and $n + m = 1111$, then $m = 555$ and $n = 556$, giving $m^2 = 308025$ and $n^2 = 309136$ and these are not four-digit squares. If $n - m = 11$ and $n + m = 101$,then $m = 45$ and $n = 56$, giving $m^2 = 2025$ and $n^2 = 3136$. These are both four-digit squares. Hence this is the only solution.

□

Problem 679

Correct solutions were submitted by Isabella Tucker, Jason Peterson Nicholas Stanlis, Ruvarashe Musasiwa, Lane Sirois, Carter Moleski and Kendal Swatosh. Based on a draw of lots, the prize is divided between Musasiwa and Swatosh. Partial solutions were also submitted by Claudia Mapes, Sydnee Renee Allen and Sungeun Kim.